CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the October/November 2014 series

4037 ADDITIONAL MATHEMATICS

4037/22 Paper 2, maximum raw mark 80

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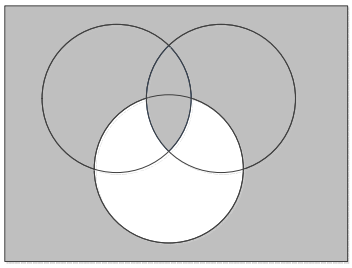
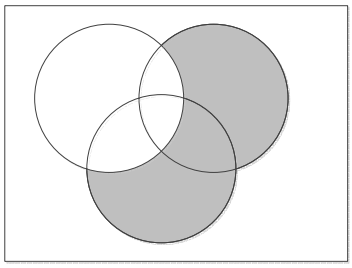
1 **(a)**		**B1**	
		**B1**	
(b)	No.in H only $= 50 - x$; No in F only $= 60 - x$ Sum: $50 - x + 60 - x + x + 30 - 2x = 98$ $x = 14$	**B1** **M1** **A1**	Both written or on diagram Add at least 3 terms each with x involved and equate to 98 soi
2	$9x^2 + 2x - 1 < (x+1)^2$ $8x^2 < 2$ oe isw $-\frac{1}{2} < x < \frac{1}{2}$	**M1** **A1** **A1**	Expand and collect terms
3	$\log_2(x+3) = \log_2 y + 2 \rightarrow x + 3 = 4y$ $\log_2(x+y) = 3 \rightarrow x + y = 8$ $x + 3 = 4(8 - x)$ $5x = 29 \rightarrow x = 5.8,$ oe $y = 2.2$ oe	**B1** **B1** **M1** **A1** **A1**	Eliminate y or x from two linear three term equations

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4	**(i)**	$\mathrm{f}(37)=3$ or $\mathrm{gf}(x)=\dfrac{\sqrt{x-1}-3-2}{2\left(\sqrt{x-1}-3\right)-3}$	**B1**	
		$\mathrm{gf}(37)=\dfrac{3-2}{6-3}=\dfrac{1}{3}$	**B1**	
	(ii)	$y=\sqrt{x-1}-3 \rightarrow (y+3)^2=x-1$	**M1**	Rearrange and square in any order
		$(x+3)^2+1=\mathrm{f}^{-1}(x)$ oe isw	**A1**	Interchange x and y and complete
	(iii)	$y=\dfrac{x-2}{2x-3}$		
		$2xy-3y=x-2 \rightarrow 2xy-x=3y-2$	**M1**	Multiply and collect like terms
		$\dfrac{3x-2}{2x-1}=\mathrm{g}^{-1}(x)$ oe	**A1**	Interchange and complete Mark final answer
5	**(i)**	$B=900$	**B1**	
	(ii)	$B=500+400\mathrm{e}^2=3455$ or 3456 or 3460	**B1**	3455.6 scores **B0**
	(iii)	$\left(\dfrac{\mathrm{d}B}{\mathrm{d}t}=\right)80\mathrm{e}^{0.2t}$	**B1**	
		$t=10 \rightarrow \dfrac{\mathrm{d}B}{\mathrm{d}t}=80\mathrm{e}^2=591\,(/\mathrm{day})$	**B1**	awrt
	(iv)	$10\,000=500+400\mathrm{e}^{0.2t} \rightarrow \mathrm{e}^{0.2t}=(23.75)$	**M1**	$\mathrm{e}^{0.2t}=k$
		$0.2t=\ln 23.75$	**DM1**	take logs: $0.2t=\ln k$
		$t=15.8\,(\mathrm{days})$	**A1**	awrt

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6 **(i)**	$(x+2)^2 + x^2 = 10$	**B1**	
	$x^2 + 2x - 3 = 0 \rightarrow (x+3)(x-1) = 0$	**M1**	3 term quadratic with attempt to solve
	Points (1, 3), (–3, –1) isw	**A1**	both x or a pair
		A1	both y or second pair
	or elimination of x leads to $y^2 - 2y - 3 = 0$, then as above		
(ii)	$m^2x^2 + 10mx + 25 + x^2 = 10$	**B1**	
	$(m^2+1)x^2 + 10mx + 15 = 0$		
	$b^2 - 4ac = (0) \rightarrow 100m^2 - 60(m^2+1) = 0$	**M1**	attempt to use discriminant on three term quadratic. Allow unsimplified
		A1	
	$m = \pm\sqrt{\frac{3}{2}}$ oe isw	**A1**	cao ± is required
	Alternative solution:		
	$\frac{dy}{dx} = \frac{-x}{\sqrt{10-x^2}}$ or $\frac{dy}{dx} = -\frac{x}{y}$	**B1**	allow unsimplified
	Result:		
	$y^2 = x^2 + 5y$ after inserted in $y = mx + 5$		
	Attempt to solve with $x^2 + y^2 = 10$	**M1**	Eliminate x or y
	$y = 2, x = \pm\sqrt{6}$	**A1**	both
	$m = \pm\frac{3}{\sqrt{6}}$ oe	**A1**	
7 **(i)**	$v = 2\cos t + 1$	**B1**	mark final answer
(ii)	$2\cos t + 1 = 0$	**M1**	equate their v to zero (must be a differential) and attempt to solve to find an **angle**
	$t = \frac{2\pi}{3}$ or 2.09	**A1**	awrt
(iii)	$t = \frac{2\pi}{3} \rightarrow x = 2\sin\left(\frac{2\pi}{3}\right) + \frac{2\pi}{3} = 3.83\,\text{m}$	**B1**	awrt
	$a = -2\sin t$	**B1ft**	ft *their* v (2nd differential)
	$t = \frac{2\pi}{3}\ a = -\sqrt{3} = -1.73$ or $-1.74\,\text{ms}^{-2}$	**DB1ft**	ft using *their* **angle** t in correct a awrt
8 **(i)**	$\frac{dy}{dx} = \frac{(2+x^2)\times 2x - x^2 \times 2x}{(2+x^2)^2} = \frac{4x}{(2+x^2)^2}$	**M1**	apply quotient or product rule
		A1	unsimplified
	$k = 4$	**A1**	k=4 does not need to be specifically identified
(ii)	$\int \frac{x}{(2+x^2)^2}\,dx = \frac{1}{4} \times \frac{x^2}{2+x^2} + (c)$ isw	**B1**	$\frac{1}{\text{their } k} \times$ original function
		B1	

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9	$\left(a+3\sqrt{5}\right)^2 = a^2 + 3\sqrt{5}a + 3\sqrt{5}a + 45$ oe	**B1**	anywhere
	Equate: $a^2 + a + 45 = 51$	**B1**	
	and $6a - b = 0$	**B1**	
	$(a+3)(a-2) = 0$	**M1**	Attempt to solve three term quadratic with integer coefficients obtained by equating coeffs
	$a = -3, 2$	**A1**	Both *a*s correct or one correct pair
	$b = -18, 12$	**A1**	Both *b*s correct
10 (i)	$\sec x \operatorname{cosec} x = \dfrac{1}{\cos x \sin x}$	**B1**	anywhere
	$\cot x = \dfrac{\cos x}{\sin x}$	**B1**	anywhere
	LHS $= \dfrac{1-\cos^2 x}{\cos x \sin x}$ oe	**B1ft**	correct addition of *their* terms
	$= \dfrac{\sin^2 x}{\cos x \sin x} = \tan x$ AG	**B1**	use of identity and cancel
(ii)	$3\cot x - \cot x = \tan x \rightarrow 2\cot x = \tan x$	**M1**	equate and collect like terms, allow sign errors
	$\tan^2 x = 2$ oe	**A1**	
	x = 54.7, 125.3, 234.7, 305.3	**A1**	2 values
		A1	only 2 more values. awrt
11 (i)	Area of sector $= \dfrac{1}{2} \times x^2 \times 0.8 \left(= 0.4x^2 \text{ cm}^2\right)$	**B1**	anywhere
	$SR = 5\sin 0.8 \left(= 3.59\right)$ or	**B1**	*SR* may be seen in stated $\dfrac{1}{2}ab\sin C$
	$OR = 5\cos 0.8 \left(= 3.48\right)$		
	Area of triangle =		
	$\dfrac{1}{2} 5\cos 0.8 \times 5\sin 0.8 = 6.247 \text{ cm}^2$	**M1**	insert correct terms into correct area formulae
	$0.08x^2 = 6.247$	**A1**	
	$x = 8.837$ cm AG	**A1**	
(ii)	$SQ = 8.84 - 5 \left(= 3.84 \text{ cm}\right)$		
	$PR = 8.84 - 5\cos 0.8 \left(= 5.35 \text{ or } 5.36 \text{ cm}\right)$	**B1**	two lengths from *SQ, PR, PQ* awrt
	$PQ = 8.84 \times 0.8 \left(= 7.07 \text{ cm}\right)$	**B1**	third length awrt
	Perimeter = 19.84 to 19.86 cm or rounded to 19.8 or 19.9	**B1**	sum
(iii)	Area $PQSR = 4 \times 6.247$	**M1**	
	$= 25 \text{ cm}^2$	**A1**	24.95 to 25

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12 **(i)**	$f(2) = 3(2^3) - 14(2^2) + 32 = 0$ Or complete long division	**B1**	
(ii)	$f(x) = (x-2)(3x^2 - 8x - 16)$	**M1** **A1**	$3x^2$ and 16 $8x$ and correct signs
	$f(x) = (x-2)(x-4)(3x+4)$	**M1** **A1**	Factorise three term quadratic
(iii)	$x = 2, 4$	**B1**	
(iv)	$\int 3x - 14 + \frac{32}{x^2}\,dx = 1.5x^2 - 14x - \frac{32}{x}(+c)$	**B1** **B1**	first 2 terms third term correct unsimplified
	Area $= \left[1.5x^2 - 14x - \frac{32}{x}\right]_2^4$	**M1**	Limits of 2 and 4 and subtract
	$= (-)\ 2$	**A1**	

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